

This article examines general properties of nonsteady solutions of nonlinear systems describing filtration or heat transfer in a heterogeneous medium with allowance for the nonlinearity of its properties.

1. Transport processes in heterogeneous media are often modeled on the basis of several coexisting homogeneous continua with heat transfer between them. Here, the structural-mechanical properties of the heterogeneous media generally depend appreciably (nonlinearly) on the transport conditions. It is interesting to study the general properties of transport equations. We do this below, using as an example the filtration of a fluid in a cracked-porous medium.

The experimental data (see [1], for example) indicates that the filtration characteristics of a medium are heavily nonlinearly dependent on its stress state. This dependence cannot be adequately accounted for within the framework of the well-known model [2], the latter in particular precluding substantial deformation of cracks and their closure. In this regard, the model of an elastic compressible cracked-porous medium in [3] is more suitable. The permeability tensor and porosity of such a medium for different stress states were determined in [4], while steady filtration was examined in [5]. Irreversible deformations of the cracks were considered in the model in [6] for closed beds. Below, we use the model in [3] to examine nonsteady filtration for the case when the cracks are open over the entire flow region. The process is described by the system of equations

$$a \frac{\partial \psi_1}{\partial t} = \frac{1}{4} \nabla^2 \psi_1^4 + \psi_2 - \psi_1, \quad \frac{\partial \psi_2}{\partial t} = \varepsilon \nabla^2 \psi_2 - \psi_2 + \psi_1, \quad (1)$$

where

$$\begin{aligned} \psi_i &= (p_i - \sigma)/(p^\circ - \sigma) \quad (i = 1, 2), \quad \xi = x/(\kappa_1 \tau)^{1/2}, \quad t = \omega/\tau; \\ a &= m_1^\circ [m_2^\circ (p^\circ - \sigma) (\beta_p + \beta_m)]^{-1}, \quad \varepsilon = h_2^\circ/h_1^\circ = \kappa_2/\kappa_1 \ll 1; \\ \kappa_2 &= h_2^\circ [m_2^\circ \mu^\circ (\beta_p + \beta_m)]^{-1}. \end{aligned} \quad (2)$$

System (1) is valid only at $p_1 > \sigma$ ($0 < \psi \leq 1$), when the cracks are open over the entire filtration region. At $p_1 \leq \sigma$, the cracks are closed and filtration in this zone occurs by blocks (elastic regime equation). Here, compatibility conditions must be satisfied on the unknown boundary. In accordance with the present formulation of the problem, below we assume that $p^\circ > p_0 > \sigma$ (the permeability of the cracks is everywhere considerably greater than the permeability of the blocks).

2. Let us examine the qualitative characteristics of model (1). First we will evaluate the coefficient a . In actuality, the difference $p^\circ - \sigma \sim (10^5 - 10^7) \text{ N/m}^2$. Taking normal values for the parameters [7] $\beta \sim (10^{-8} - 10^{-9}) \text{ m}^2/\text{N}$, $\beta_m \sim 10^{-9} \text{ m}^2/\text{N}$, $m_1^\circ/m_2^\circ \sim 10^{-1}$ ($r = 1, 2, \dots$), we find $a \sim (10^{1-r} - 10^{3p-r})$. Thus, in contrast to the model in [2], the term $a \partial \psi_1 / \partial t$ in the first equation of (1) cannot be ignored in the general case.

An important feature of model (1) is the presence of nonlinearity, characterizing the elastic deformation of the cracks (the corresponding expression for filtration velocity can be found in the literature; see [8], for example).

The role of the coefficient a and the nonlinear term in Eq. (1) can be evaluated, following [1], by examining the propagation of a perturbation introduced into the system at $\omega = x =$

0. To do this, we replace the second equation by the sum of Eqs. (1) and we insert it into the new system

$$\psi_i = n_i \bar{\psi}_i, \quad t = k\bar{t}, \quad \xi = l\bar{\xi}, \quad \bar{\psi}_i \sim \bar{t} \sim \bar{\xi} \sim 1,$$

where n_i are characteristic changes in pressure ψ_i in the region of a bed with a linear scale l ; k is the corresponding time scale. We obtain

$$\begin{aligned} \frac{\partial \bar{\psi}_1}{\partial \bar{t}} &= \frac{n_1^3 k}{4al^2} \nabla^2 \bar{\psi}_1^4 + \frac{k}{a} \left(\frac{n_2}{n_1} \bar{\psi}_2 - \bar{\psi}_1 \right), \\ \frac{\partial \bar{\psi}_2}{\partial \bar{t}} &= \frac{an_1}{n_2} \frac{\partial \bar{\psi}_1}{\partial \bar{t}} = \frac{k}{l^2} \left(\varepsilon \nabla^2 \bar{\psi}_2 + \frac{n_1^4}{4n_2} \bar{\psi}_1^4 \right). \end{aligned} \quad (3)$$

Let $a \sim \varepsilon$ (which corresponds to the model in [2] "to within the nonlinearity"). Evaluating the terms in (3) at $n_2/n_1 \sim \varepsilon$ (the pressure changes significantly in the cracks compared to the blocks), we find that this process takes place over a short period of time $k \sim \varepsilon$ ($\omega \sim \varepsilon\tau \ll \tau$ in dimensional form) in the region

$$4l^2 \sim n_1^3 \quad (4x^2 \sim n_1^3 \kappa_1 \tau) \quad (4)$$

and is described by the equations

$$\frac{a \partial \bar{\psi}_1}{\partial \bar{t}} = \frac{1}{4} \nabla^2 \bar{\psi}_1^4 - \bar{\psi}_1, \quad \frac{\partial \bar{\psi}_2}{\partial \bar{t}} = \bar{\psi}_1. \quad (5)$$

At this stage, the coefficient analogous to pressure conduction in linear filtration is equal to $1/(4a) \gg 1$ (rapid motion), and the fluid flows into the blocks.

It is similarly established that filtration with $n_2/n_1 \sim 1$ (pressure changes of the same order of magnitude in the cracks and blocks) occurs in region (4) at $k \sim 1$ ($\omega \sim \tau$) and is described by the equations

$$\frac{1}{4} \nabla^2 \bar{\psi}_1^4 + \bar{\psi}_2 - \bar{\psi}_1 = 0, \quad \frac{\partial \bar{\psi}_2}{\partial \bar{t}} = \frac{1}{4} \nabla^2 \bar{\psi}_1^4, \quad (6)$$

the "pressure conduction coefficient" now being equal to $1/4$ (determined by the permeability of the cracks and the compressibility of the blocks, motion being slower in this case).

Thus, the qualitative development of filtration processes with $n_2/n_1 \sim \varepsilon$ and $n_2/n_1 \sim 1$ is similar for model (1) at $a \sim \varepsilon$ and model [2] (Eqs. (5), (6) and (22.11), (22.5) in [1] coincide "to within the nonlinearity").

At $a \sim 1$ and $k \sim 1$, filtration occurs with $n_2/n_1 \sim 1$ in region (4) (the process with $n_2/n_1 \sim \varepsilon$ is now impossible in the same region):

$$a \frac{\partial \bar{\psi}_1}{\partial \bar{t}} = \frac{1}{4} \nabla^2 \bar{\psi}_1^4 + \bar{\psi}_2 - \bar{\psi}_1, \quad \frac{\partial \bar{\psi}_2}{\partial \bar{t}} = \bar{\psi}_1 - \bar{\psi}_2. \quad (7)$$

Compared to (5), the blocks now affect the pressure distribution in the cracks, since filtration occurs slowly. The process is established in region (4) at $k \sim \varepsilon^{-1}$:

$$\nabla^2 \bar{\psi}_i^4 = 0 \quad (i = 1, 2). \quad (8)$$

With a further increase in the parameter a ($a \sim \varepsilon^{-1}$), filtration slows even more. In region (4) at $k \sim 1$, the system still does not "sense" the perturbation, while at $k \sim \varepsilon^{-1}$ the process is described by the equations

$$a \frac{\partial \bar{\psi}_i}{\partial \bar{t}} = \frac{1}{4} \nabla^2 \bar{\psi}_i^4 \quad (i = 1, 2),$$

with a stationary pressure distribution (8) being established at $k \sim \varepsilon^{-2}$.

Analysis of Eqs. (3) shows that the effect of the nonlinear term nearly precludes filtration with $n_1/n_2 \sim \varepsilon$ (the pressure changes in the blocks are much greater than in the cracks) at values of $a \sim (\varepsilon, 1, \varepsilon^{-1})$. For the model in [2], this stage occurs at $k \sim 1$ in the region $l^2 \sim \varepsilon$.

At surfaces of discontinuity, the same conditions are present as for the model [2] (are established analogously to [2]): ψ_1 , $\partial\psi_1/\partial\xi$ are continuous; ψ_2 , $\partial\psi_2/\partial\xi$ are continuous at $\varepsilon \neq 0$, while at $\varepsilon = 0$ the discontinuities decay exponentially over time.

Integrating Eqs. (1) over time for $0 \leq t \leq s \leq 1$, we find $\psi_i(x, s) \rightarrow \psi_i(x, 0)$ at $s \rightarrow 0$. Thus, the initial values of the functions ψ_i can be prescribed independently (for model [2], this is possible only with allowance for the volume of the cracks, which is usually much less than the volume of the pores). Excluding one of the unknown functions in (1), we find that the effective equations for ψ_1 , ψ_2 will be different, and they will coincide only at $\tau \rightarrow 0$,

$$\frac{\partial\psi}{\partial\omega} = \nabla \left(\frac{\kappa_2 + \kappa_1\psi^3}{1+a} \nabla\psi \right) \quad (9)$$

(the medium becomes porous, with an overall pressure conduction coefficient). For the model [2], the effective equations coincide at $m_1^0 = h_2^0 = 0$.

3. Let us begin to construct the solution of system (1). After a critical evaluation of the well-known methods of solving nonlinear systems with partial derivatives, we chose the method of integral relations [2] as the most effective and simplest method. Physically clear conclusions can be drawn even in the first approximation, corresponding to the method of successive substitution of steady states.

Changing over to zero boundary conditions, we will examine the filtration of a fluid to a tunnel:

$$\frac{a\partial\psi_1}{\partial t} = \frac{1}{4} \frac{\partial^2(\psi_1 + 1)^4}{\partial\xi^2} + \psi_2 - \psi_1, \quad \frac{\partial\psi_2}{\partial t} = \varepsilon \frac{\partial^2\psi_2}{\partial\xi^2} - \psi_2 + \psi_1, \quad (10)$$

$$\psi_i = (p_i - p^0)/(p^0 - \sigma) \quad (i = 1, 2).$$

In accordance with the chosen method, we seek the solution of (10) in the form [2, 5]

$$\psi_i(\xi, t) = -1 + |\alpha + (1 - \alpha)\xi/l_i(t)|^{1/4} \quad (i = 1, 2), \quad \alpha = (1 - \nu)^4, \quad (11)$$

satisfying the boundary conditions

$$\psi_i(0, t) = -\nu = -(p^0 - p_0)/(p^0 - \sigma) \quad (p_i(0, t) = p_0), \quad (12)$$

$$\psi_i(l_i, t) = 0 \quad (p_i(l_i, t) = p^0).$$

In (11), we introduced two boundaries $l_i = l_i(t)$ of perturbation zones propagating through the cracks and blocks with the start-up of the tunnel [9].

The laws of motion of $l_i = l_i(t)$ are found from the second integral relations corresponding to (10):

$$l_1^2(t) = \frac{1}{1+a} \left[\frac{A-aB}{1+a} (1 - \exp(rt)) + (A+B)t \right],$$

$$l_2^2(t) = \frac{1}{1+a} \left[-\frac{a(A-aB)}{1+a} (1 - \exp(rt)) + (A+B)t \right], \quad (13)$$

$$A = \frac{\alpha-1}{4\beta}, \quad B = -\frac{\varepsilon\nu}{\beta}, \quad \beta = -\frac{1}{2} + \frac{4(5-9\alpha+4\alpha^{9/4})}{45(1-\alpha)^2}, \quad r = -1 - \frac{1}{a}.$$

The functions $A = A(\nu)$, $B = B(\nu)$ are monotonic for $0 < \nu < 1$, $4.5 < A(\nu) < 6$, $6\varepsilon < B(\nu) < 18\varepsilon$.

It follows from (13) for sufficiently large values of time

$$l_1^2(t) \approx l_2^2(t) \approx (A+B)t/(1+a) \quad (14)$$

that the filtration process takes place as it does in a normal porous medium. The case $l_1(t)$ $l_2(t)$ for any t is examined below. Here, we assume that $l_1(t) \neq l_2(t)$ for finite t ($t \neq 0$).

The first phase of the process ends at the moment $t = t_1$, when the boundary l_1 reaches the contour of the bed (half the distance to the adjacent well):

$$l_1^2(t_1) = L^2. \quad (15)$$

The solution of system (10) is similarly constructed for the second phase and has the form*

$$\begin{aligned} \psi_1(\xi) = -1 + [\alpha + (1 - \alpha)\xi/L]^{1/4}, \quad \psi_2(\xi, t) = -1 + [\alpha + (1 - \alpha)\xi/l_2(t)]^{1/4}, \\ l_2^2(t) = l_{22}^2(t) = \gamma + [l_{21}^2(t_1) - \gamma] \exp(t_1 - t), \quad \gamma = L^2 - \xi v/\beta, \end{aligned} \quad (16)$$

where the second subscript in the expression $l_{2i}(t)$ indicates the phase of the process ($l_{21}^2(t_1)$ is also determined by the second formula in (13)).

The filtration process is established at the moment $t = t_2$, when the boundary $l_2(t)$ reaches the position L:

$$l_{22}^2(t_2) = L^2, \quad (17)$$

and the functions $\psi_i(\xi)$ have the form [5]:

$$\psi_1(\xi) = \psi_2(\xi) = -1 + [\alpha + (1 - \alpha)\xi/L]^{1/4}. \quad (18)$$

We use (17) to find the time required to establish the process

$$t_2(a, L, v) = t_1 + \ln[1 + \delta(t_1)], \quad \delta(t_1) = \frac{A - ab}{B(1+a)} [1 - \exp(rt_1)], \quad (19)$$

where the moment of time $t = t_1$ is determined from Eq. (15):

$$B\delta(t_1) + (A + B)t_1 = (1 + a)L^2. \quad (20)$$

Using (19) and (20) and experimental data on filtration, we can evaluate the parameter a for the reservoir being examined here.

At $a = A/B$ for all values of t , the laws of motion of $l_i = l_i(t)$ coincide with (14). In this case

$$\delta(t_1) = 0, \quad t_2 = t_1 = (1 + a)L^2/(A + B).$$

4. The effect of the dimensionless complexes a , L , and v on the time of establishment of the process can be evaluated from Eq. (20), taking into account that it follows from (19) that $dt_2/dt_1 > 0$. For example, differentiating (20) with respect to a (assuming $t_1 = t_1(a)$), we find $dt_1/da > 0$. Thus, the transient period $t = t_2$ increases with an increase in the parameter a . We similarly obtain $dt_2/dL^2 > 0$, $dt_2/dv > 0$. Meanwhile, the effect of the complex v (in particular, the depression) is fairly weak (the coefficients A and B used to express v in (19) and (20) change little).

Let us evaluate the order of the parameter L (the order of the parameter a was evaluated above). For $100 \text{ m} < X < 2000 \text{ m}$ and pressure conduction of $0.1 \text{ m}^2/\text{sec} \leq \kappa_1 \leq 5 \text{ m}^2/\text{sec}$, we obtain $L^2 \sim (10^4 - 10^7) \tau^{-1}$ ($[\tau] = \text{c}$). The lag time is determined from the pressure recovery [10] and varies broadly for different beds (from several minutes to several hours), amounting to 15-20% of the transient period. For example, $L^2 \sim (1 - 10^4)$ for $\tau = 10 \text{ min}$, while $L^2 \sim (10^{-1} - 10^3)$ for $\tau = 5 \text{ h}$.

In the special cases $a \ll 1$, $a \gg 1$, Eqs. (19) and (20) assume a simpler form. At $a \ll 1$

$$\delta(t_1) \approx \frac{A}{B} \left[1 - \exp\left(-\frac{t_1}{a}\right) \right] \approx 0.6e^{-1} \left[1 - \exp\left(-\frac{t_1}{a}\right) \right],$$

where the value $t = t_1$ is determined from the equation

$$0.2L^2 - 1 + \exp(-t_1/a) - t_1 = 0.$$

At $a \gg 1$ $\delta(t_1) \approx (-1 + A/aB)[1 - \exp(-t_1)]$ and $B\delta(t_1) < 1$. If $aL^2 \gg 1$ in (20), then $t_1 \approx$

*The solution (16), characterizing an intermediate phase of the process, is somewhat conditional in character (flow has already been established in the cracks but not in the blocks) due to the specific features of the method of successive substitution of steady states.

TABLE 1. Range of the Transient Period of the Filtration Process at $100 \text{ m} \leq X \leq 2000 \text{ m}$

$\tau, 10^{-4}$ sec	$\kappa_1, \text{m}^2/\text{sec}$					
	1		3		1	
	$a=0,1$		$a=1$		$a=10$	
0,06	1	0,7	16,5	0,8	6	2,2
	9,3	3,1	17,4	5,9	92,5	30,9
0,18	2,1	1,5	2,4	1,6	6,4	2,7
	9,3	3,1	17,5	5,9	92,5	30,9
0,36	3,1	2,3	3,5	2,4	6,8	3,4
	9,4	3,2	17,6	5,9	92,5	30,9
1,8	9,7	4,7	9,8	4,8	12,7	6,2
	9,8	3,7	18,2	6,2	92,5	30,9

Note: The transient period for each lag is given in hours in the first row and days in the second row.

TABLE 2. Comparison of Transient Periods of the Filtration Process in an Elastic Compressible Cracked-Porous Reservoir at $X = 300 \text{ m}$, $\kappa = 1 \text{ m}^2/\text{sec}$

$\tau, 10^{-4}$ sec	ω			Porous medium
	$a=0,1$	$a=1$	$a=10$	
	h		days	
0,06	5,5	10	2,1	Transient period 3 h [2]
0,18	6,5	10,7	2,12	
0,36	8,1	12,1	2,16	
1,8	20	22	2,45	

$\alpha L^2 / (A + B) \approx 0.2\alpha L^2$. The condition $\alpha L^2 \gg 1$ means that there is the following limitation on τ ,

$$\tau \ll \frac{\alpha X^2}{\kappa_1} \quad (21)$$

For example, at $\alpha = 1$, $X = 500 \text{ m}$, and $\kappa_1 = 5 \text{ m}^2/\text{sec}$, we obtain $\tau \ll 14 \text{ h}$. Thus, condition (21) is realistic.

The transient period calculated from Eqs. (19) and (20) is shown in Table 1. Analysis of Eqs. (19) and (20) showed that ϵ has little effect on the transient period, so we took a fixed value $\epsilon = 10^{-2}$ in the calculations. For comparison, Table 2 also indicates the transient period for model (1) and the porous medium.

The following conclusions can be made from the calculated results.

The dimension X , the pressure conduction of the cracks, and the parameter α have a very large effect on the transient period. A lag is manifest to the greatest extent with small values of crack pressure conduction and small values of X . The effect of τ on the transient period decreases with an increase in the parameter α . For the investigated values $\alpha \geq 0.1$, the process of filtration in an elastic compressible cracked-porous reservoir is established much more slowly than in a porous medium. When the more approximate formula $t_1 \approx 0.2\alpha L^2$ is used (for $\alpha = 10$), the error is no greater than 15%.

It should be noted that a rough estimate of the transient period can be obtained as follows for $L^2 \gg 1$. In (20), the term $B\delta(t_1)$ is finite at any α and $t_1 \approx 0.2(1 + \alpha)L^2$. At $|rt_1| \geq 2$ $\exp(rt_1) \leq 0.13$ and $\delta(t_1) \approx (A - \alpha B) / [B(1 + \alpha)]$.

5. Let us examine the effect of the parameters on the yield q .

In the first phase, with allowance for (13)

$$q(t, v, a, \epsilon) = \frac{1}{4} \left[\frac{\partial(\psi_1 + 1)^4}{\partial \xi} + \epsilon \frac{\partial \psi_2}{\partial \xi} \right]_{\xi=0} = \frac{1 - \alpha}{4} \left[\frac{1}{l_1(t)} + \frac{\epsilon}{\alpha^{3/4} l_2(t)} \right]$$

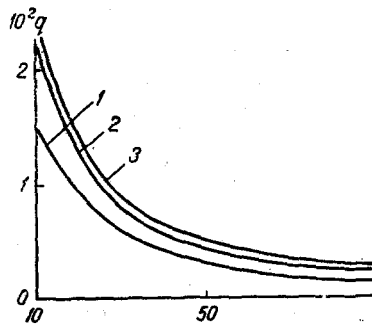


Fig. 1

Fig. 1. Dependence of the yield on the parameter $L = X / (\kappa_1 \tau)^{1/2}$: 1) $\nu = 0.2$; 2) 0.4; 3) 0.8.

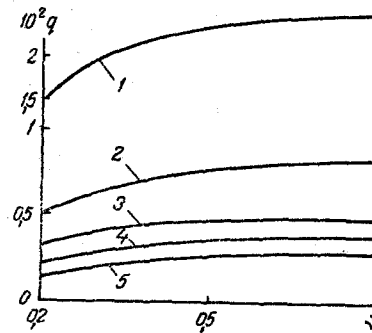


Fig. 2

Fig. 2. Dependence of the yield on the parameter $\nu = (p^\circ - p_0) / (p^\circ - \sigma)$: 1) $L = 10$; 2) 30; 3) 50; 4) 70; 5) 100.

the yield increases with an increase in the parameters α and ν and decreases with the passage of time ($\partial z_1 / \partial \alpha < 0$, $\partial z_1 / \partial \nu < 0$). In the second phase

$$q(t, \nu, \alpha, \varepsilon, L) = \frac{1-\alpha}{4} \left(\frac{1}{L} + \frac{\varepsilon}{\alpha^{3/4} l_2(t)} \right)$$

the values of α and t have very little effect (filtration takes place only through the blocks), while the yield now increases more slowly with an increase in ν .

In the third phase

$$q(\nu, \alpha, \varepsilon, L) = \frac{1-\alpha}{4L} \left(1 + \frac{\varepsilon}{\alpha^{3/4}} \right) \approx \frac{1-\alpha}{4L}$$

there is almost no change in the dependence of the yield on the parameters ν and α . The yield decreases with an increase in L in the second and third phases, which signifies a redistribution of the fixed depression over a greater distance.

On the whole, a change in the parameters affects the yield mainly through the cracks. The final yield has practically been established by the end of the first phase.

The conclusions reached here (and illustrated by Figs. 1 and 2, where all of the parameters are dimensionless) are consistent with the analysis of the steady-state case in [5]: the yield increases as the cracks are compressed (as ν increases) and nearly stabilizes when a region of closed cracks develops near the tunnel.

Thus, the above-examined example of filtration in cracked-porous materials shows the strong dependence of transport processes in heterogeneous media on changes in the structural-mechanical properties.

NOTATION

p , x , ω and ψ , ξ , t , dimensional and dimensionless pressure, coordinate, and time, respectively; m , h , κ , porosity, permeability, and pressure conduction; β_p and β_m , coefficients of compressibility of the fluid and the pores in the blocks; μ , viscosity of the fluid; τ , lag time; p° , p_0 , σ , characteristic values of pressure; $l(t)$, boundary of perturbation zone; q , yield; n , l , k , scales of pressure, bed region, and time; α , ε , L , and ν , dimensionless parameters; the symbol f° denotes characteristics of the system with an initial bed pressure p° . Indices: 1, 2, cracks and blocks, respectively.

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PARAMETERS OF WAVE INSTABILITY IN A BOUNDARY LAYER

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The waves developing in a boundary layer during transition from laminar to turbulent flow are investigated experimentally.

The experimental study of flow in a boundary layer of a wing in an aerodynamic wind tunnel and under natural conditions, i.e., in flight (including in clouds and near the earth), has shown [1] that in all these cases the transition from laminar to turbulent flow occurs through the development of an instability wave packet in the region of a positive pressure gradient, despite the different level and frequency composition of the leading flow turbulence. The mean frequency of this wave packet depends on the flow velocity and on the angle of attack.

Quite important and demanding solution is the problem of which factors determine the frequency and wavelength in each specific case. Using this as a starting point, the purpose of the present study has been to explain the general features of the parameters of waves, generated on the same profile for different attack angles of the model and flow velocities in the tube. To solve this problem in an aerodynamic wind tunnel, experimental measurements of wave instability were carried out in a boundary layer on a wing model in a wide range of flow regimes, and then were calculated and analyzed the dimensionless parameters describing this effect, including the known frequency parameter $F = 2\pi f v / U_\infty^2$ (see [2]), and the parameters $2\pi f \delta_1 / U_\infty$ and $2\pi \delta / \lambda$, used theoretically [3].

The studies were carried out in the aerodynamic wind tunnel T-324 of the Institute of Theoretical and Applied Mechanics, Siberian Branch of the Academy of Sciences of the USSR, having a degree of flow turbulence less than 0.04% [4]. The model of the wing, made of wood and coated varnish, had a NACA 63-2-615 profile. The wingspan of the model was 1 m, and the mean chord, along which measurements were taken, was $b = 0.27$ m. Ten drainage points were used to measure the static pressure at the upper surface of the model. The model was set up vertically in the operating portion of a tube with a square cross section. The angle of attack was determined relatively to the walls of the operating portion. The experiments were carried out for attack angles of the model of -4 , 0 , and 4° for a leading flow velocity of 25 m/sec and in the flow velocity region of 10-40 m/sec at an attack angle of the model being 4° .

The thermoanemometric complex DISA 55D was used for measurements in the boundary layer. The detector of the thermoanemometer with a filament of diameter of 6 μm and length of 2 mm was attached to the support, coated inside the tube by a fairing so as to reduce vibrations. The support was fixed in a coordinate grid, established in the window of the operating portion, and allowing to displace the detector with an accuracy of ± 0.5 mm in the longitudinal x direction and with an accuracy of ± 0.01 mm in the transverse y direction. To guarantee reliable determination of the moment the detector touches the surface of the model, which is necessary for correct determination of the boundary layer thickness, the model was rubbed with

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